



Qualitative Study on an Improved Fuzzy-Based Service Rates' Control System for ... Gbadebo, A. D., Et. al. (2026)

Qualitative Study on an Improved Fuzzy-Based Service Rates' Control System for Single Server Queue Networks with Vacations: A Review

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Abstract

The task of defining service rates in queue networks is a dimensional problem as it revolves around network properties and customers' behaviour. We examined a fuzzy-based control system for a one-server queue system with possible vacations. It is an optimal system such that arrivals which find customers awaiting service turns are rejected. Our fuzzy-based model dynamically select service rates considering system's properties in order to minimize cost. We simulated the system beginning with initial condition where all crisp inputs are zero. This hysteretic property indicates that the server is switched on for a 2,500 time units, thereby indicating that the optimal policy is an hysteretic property of s.

Keywords: Admission, costs, optimality, vacations, service requests, hysteretic property

Introduction

Queues are common events in daily lives. People queue in supermarkets, banks, service stations, hospitals, etc. While some queues are visible, others are not. Queues of data packets as well as voice calls in communication channels are, for instance invisible. In most cases, it is undesirable to have queues as a result of associated costs of time, network resources, etc. However, queues emerge as a result of inadequate service sources to manage existing service requests (Abdali et al. 2024). This often results from the fact that servers may be unavailable because of space or cost limitations, or the system is unable to provide the level of service necessary to prevent waiting. A queue is a stochastic system involving arrivals which wait in queue until they are served by one or more servers and consequent departures. A queue system is often defined by its servers, service type and the amount of customers awaiting service turns in each queue. In a queue system with variable rate where mean rate of service is selected among service rates of:

$$\{\mu_k \mid k \in K\}, \quad K \text{ is defined as fixed service}$$

type, while service time of an arrival is a stochastic value. It is a multi-dimensional optimal problem to determine the time to switch on or off a server. In order to do this successfully, it is important to know the cost benefit for switching on or off server while juxtaposing this with the associated cost of service to customers waiting to be served. According to Badian-Pessot, Lewis and Down (2019), it is important to observe that while servers are on, it is possible to adopt a flexible service rate which is



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optimal in nature. A service policy is said to be of an N -policy type if it requires switching server on with N jobs in the network while switching off the server whenever the system becomes empty. Generally, N -policies are often optimal for most $M/M/1$ networks involving initializing costs (Singh, Jain & Dhibar, 2023). Regarding $M/M/1$ networks involving flexible control of service rate, Divya and Indhira (2023) showed the optimal service rate structure is monotonic in nature. Researches on modeling of fuzzy system aimed at optimal service rates with the goal of optimizing systems' throughput had not been sufficiently covered in literature. In essence, this study gives an optimal model to enhance systems' throughput while costs relating to wait times are minimized. This optimizes performance in terms of network throughput while also ensuring prompt service for customers awaiting service turns in the system.

Related Studies

Studies on queueing system involving removable servers include Ahuja, Jain and Jain (2021) which studied $M/G/1$ systems with N -policies. This study examined the relationships between server's characteristics and admission systems. Optimality in N -policies involving $M/G/1$ systems having initialization cost with average and discounted reward system was evaluated by Wu (2025). In this study, the effects of discounted policies on network costs were evaluated. An experimental study on $M/M/1$ systems with N -policies accruing initialization as well as stochastically distributed warm-up costs was conducted by Keerthika, Niranjana and Durga (2023). However, this study did not consider service rate control of the system. Abu-Shareha, et al (2025) extended this study by including parameters of general warm-up costs thereby providing probabilities of the steady-state values, average wait time and number of customers in queue. A study on $M/G/1$ system where the service time is known at arrival was studied by Nishad, Sinha and Verma (2025). Similar studies for these network models may include networks having possible servers' breakdowns as well as initialization costs. In relation to this, queues with server vacations was investigated by Saini, Gupta and Tripathi (2023) while queues with batch arrivals were studied by Abdali, Heidari and Alipour-Vaezi (2024), among others.

A queue system in which service rates are taken from among a finite subset of the form $[0, \infty]$ was modeled by Agarwal (2023). Kumar, Lewis and Topaloglu (2023) modeled a single server network having flexible service rate control and Markov arrivals. An investigation on the state of flexible service rate vis-a-vis admission management in $M/M/1$ networks was conducted by Dimitrakopoulos and Burnetas (2017). The application of sleep-state control under various policies in $M/G/1$ queues was studied by Gebrehiwot, Aalto and Lassila (2014). Related to this Gandhi, et al (2024) examined an $M/G/1$ queue with initialization costs and idle states given specific cost value with mean feedback time in a steady state. Other studies are concerned with multi-server systems such as Gandhi, Harchol-Balter and Adan (2025), Maccio and Down (2025) as well as Maccio and Down (2025). In a research involving single server queue system by Yang and Chen (2018), a relationship between network idleness and feedbacks of the Bernoulli form was established. Similarly, Vijayalakshmi and Kalidass (2018) studied the steady-state behaviour of Markovian system having finite capacity, service breakdowns and consequent vacations. An $M/M/1$ queue model having an optional service system such that the server is subjected to service interruptions was investigated by Dong-Yuh and Yi-Hsuan (2018). Related to this, Vijayalakshmi, Kalidass and Pavithra (2018) considered an $M/M/1/N$ queue system with service breakdown and two-tier service system. In real life queue systems, it is often assumed that a server stops service entirely during the breakdown time. It is possible for a server in a breakdown state to provide services at a slower rate than its usual capacity instead of entirely stopping services during this breakdown time. This is often called "working



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breakdowns". This idea was first mentioned by Kalidass and Kasturi (2012). Since then, the concept had been studied in steady-state analysis of $M/M/1$ systems having working breakdowns. A single server queue model with working breakdown was investigated by Kim and Lee (2024). This involved a system in which service times is defined with a general distribution. Liou (2023) applied the matrix-geometric approach to study a Markovian queue of finite capacity with unreliable service system which is subject to working breakdowns and impatient customers. An analysis of steady-state analysis of $M/M/1/N$ queue with working breakdowns and feedbacks of Bernoulli form was investigated by Kalidass and Pavithra (2016).

In Nikolaev, et al (2026) the researchers developed a model of a DECT-2020 network system which was depicted as a two-tier tandem queue enabling Peak Age of Internet (PAoI) description using Laplace-Stieltjes transforms (LST). The researchers applied the dependence between packets' generation and travel times. This facilitated the derivation of LST while suggesting the probability density value for an $M/M/1$ system. Related to this, Qasim and Hamid (2024) conducted a comparative analysis of performance of traditional and AI-driven active queue approaches. It was discovered that AI-driven active queue methods performed favourably better than the traditional active queue methods. While the AI-driven active queue approaches were adaptive and flexible in queue network analysis, the traditional active queue methods were not. A study of transformative parameters for enhancing system performance of queue systems by adopting AI-based tandem analysis was conducted by Keerthika et al (2025). This approach reduced packets' waiting times and optimization of network resources, thereby finding its place in industrial applications. In a related study, Wu (2025) introduced a generalized priority queueing (GPQ) strategy, which was modeled as an advancement to first-in-first-out based routing methods. It included dynamic priority methods in the routing layer of the network thereby enabling the routing parameter to prioritize packets adaptively in the network. An assessment of existing bandwidth optimization systems was conducted by Dokic and Dokic (2025). Each model was analyzed using corresponding algorithms while suggesting other optimization models that could solve the problems of existing techniques. Related to this, Dudin, Dudin and Dudina (2026) conducted a study involving forecasting and optimization of the performance of a multiple server network with two distinct servers. The dynamics of the proposed model were characterized using multiple dimensional Markov chain describing the ergodicity of the system.

Materials and Methods

The materials and methods applied in the study are discussed in this section.

a. The analytical operations of $M/M/1$ queue networks

Assume that the amount of arrivals awaiting service turns in the queue network at time t is denoted as $N(t)$, then the dual-variate procedures of $\{(C(t), N(t)), t \geq 0\}$ can be defined as a Markov chain of the continuous time form. Consequently:

$$P_i, n(t) = \text{Pr ob}\{C(t) = i, N(t) = n\}, \quad i = 1, 2, 3, 4 \quad \text{while } n \geq 0$$

The state probability equations as provided by Vijayalakshmi, Kalidass and Deepa (2021) are:

$$\begin{aligned} \mu p P_{1,1} + \mu_1 P_{2,1} &= \lambda P_{1,0}; \\ \lambda P_{1,n-1} + \gamma P_{3,n} + \mu p P_{1,n+1} + \mu_1 P_{2,n+1} &= (\lambda + \beta + \mu) P_{1,n}, 1 \leq n \leq N-1; \quad \lambda P_{1,n-1} + \gamma P_{3,n} = (\beta + \mu) P_{1,n}, n = N; \\ \mu q P_{1,1} + \xi P_{4,1} &= (\mu_1 + \alpha + \lambda) P_{2,1} \quad \lambda P_{2,n-1} + \mu q P_{1,n} + \xi P_{4,n} = (\mu_1 + \alpha + \lambda) P_{2,n} + 2 \leq n \leq N; \\ \lambda P_{2,n-1} + \mu q P_{1,n} + \xi P_{4,n} &= (\mu_1 + \alpha) P_{2,n}, n = N; \\ \mu_b p P_{3,1} + \mu_{b1} P_{4,1} &= \lambda P_{3,0}; \end{aligned}$$

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$$\begin{aligned} \mu_1 p P_{3,n+1} + \lambda P_{3,n-1} + \beta P_{1,n} + \mu b_1 P_{4,n+1} &= (\gamma + \lambda + \mu_1) P_{3,n}, 1 \leq n \leq N-1; \\ \lambda P_{3,n-1} + \beta P_{1,n} &= (\gamma + \mu_1) P_{3,n}, n = N; \\ \alpha P_{2,1} + \mu_1 q P_{3,1} &= (\xi + \lambda + \mu b_1) P_{4,1}; \\ \alpha P_{2,N} + \mu_1 q P_{3,N} + \lambda P_{4,N-1} &= (\xi + \lambda + \mu b_1) P_{4,n}, 2 \leq n \leq N; \\ \alpha P_{2,n} + \mu_1 q P_{3,n} + \lambda P_{4,n-1} &= (\xi + \mu b_1) P_{4,n}, n = N. \end{aligned}$$

From the normalizing condition,

$$P_{1,0} + P_{1,e} + P_{2,0} + P_{2,e} + P_{3,0} + P_{3,e} + P_{4,0} + P_{4,e} = 1$$

In this case, **e** denotes column value with N dimension giving:

$$P_{1,0} = 1/\psi$$

while:

$$\Psi = 1 - A_{12} A_{22}^{-1} e + \Theta_7 A_{62} A_{22}^{-1} e + \Theta_8 e + \Theta_7 e + \Theta_5 \Theta_6^{-1} e$$

In this case, the Markov chain of the form $X_t, t \in (-\infty, \infty)$ is a stochastic process of the continuous-

time which generates the set of states $\{S_1, S_2, \dots\}$ while also satisfying a Markov notation of the

form:

$$P(X_m, | X_{m-1}, X_{m-2}, \dots, X_{t(0)}) = P(X_m, | X_{m-1})$$

where

$$-\alpha < t_0 < t_1 < \dots < t_n < \alpha$$

b. Problem definition

Figure 1 depicts a queuing network such that it is possible to switch off the server and consequently goes into vacations.

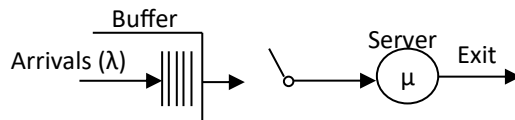


Figure 1: A queue network having a single service system with possible vacations

Arrivals into the network which has an infinite capacity is in accordance to a Poisson form with parameter λ . The network comprises of one exponential server in with service rate μ , while $\mu > \lambda$. The network is such that the service rate can be made nil at intervals based on varying network features. This network comprises of three costs viz:

- a. The switch-on cost i.e. R : This refers to cost incurred once the server is turned on. It is often depicted using: $R_k, k = 0, 1$, for each unit time;
- b. The customer servicing cost i.e. r : This refers to cost incurred as the service provides services to customers in turns. It is often expressed as: r_k , while $k = 0, 1$, per unit time; and
- a. The holding cost i.e. h : This refers to the cumulative cost per time unit for customer in the network.

The aim is arriving at a control policy that is optimal. This is a semi-Markov decision process as a result of the presence of the controller. Consequently, the times between successive service completions are no longer exponentially spread. Singh, Jain and Dhivar (2023) are of the opinion that it is optimal to keep the server always on when:

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$$\left[\frac{2r(1-\rho)}{h} + 1 \right]^2 - \frac{8\lambda R(1-\rho)}{h} < 0 \dots\dots\dots(1)$$

where $\rho = \lambda / \mu$ depicts intensity of network traffic. It is optimal to switch on the server when N customers waiting to be served. Similar to this, it is optimal to turn off the server while there no customer to be served. In this case, the least cost is given as:

$$n^* = \sqrt{\frac{2\lambda R(1-\rho)}{h}} \dots\dots\dots(2)$$

In this case, $N = 0$ when (1) is true, it is optimal to keep the server on. In this instance, the cost of switching off the server is larger than the service cost.

In the model proposed, the service rates and inter-arrival times of customers are exponential stochastic values of mean values $1/\mu$ and $1/\lambda$ respectively. In this case, the optimal control is of threshold value, implying arrivals which find K customers in queue and in service are denied admission into the system for given threshold value. The corresponding state transitions in this instance are given in figure 2.

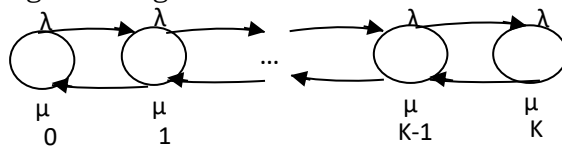


Figure 2: State transitions of the queue model

The transition rates for each case are given as follows:

$$\Lambda_{n,n+1} = \begin{cases} \lambda & \text{if } n < K, \\ 0 & \text{if } n = K. \end{cases}$$

$$\Lambda_{n,n-1} = \begin{cases} \mu & \text{if } n > 0, \\ 0 & \text{if } n = 0. \end{cases}$$

$$\Lambda_n = \begin{cases} \lambda & \text{if } n = 0, \\ \lambda + \mu & \text{if } 0 < n < K, \\ \mu & \text{if } n = K. \end{cases}$$

Transition from i to j states require j arrivals at service interval. Transition through i to j states where $j \geq i$, $-1 > 0$ requires that $(j-i+1)$ arrivals during the service interval. Similar to this, transition from i to j where $j < i-1$ is not feasible since service occurs only at given intervals

If the transition probability matrix of the fuzzy Markov chain is denoted by $\tilde{P} = [P_{ij}]$, then all $j \geq i-1$,

$i > 1$ can be represented as follows:



$$\tilde{P} = [P_{ij}] = \begin{bmatrix} \tilde{P}_0, & \tilde{P}_1, & \tilde{P}_2, & \tilde{P}_3, & \dots \\ \tilde{P}_0, & \tilde{P}_1, & \tilde{P}_2, & \tilde{P}_3, & \dots \\ 0, & \tilde{P}_0, & \tilde{P}_1, & \tilde{P}_2, & \dots \\ 0, & 0, & \tilde{P}_0, & \tilde{P}_1, & \dots \\ \cdot & \cdot & \cdot & \cdot & \dots \\ \cdot & \cdot & \cdot & \cdot & \dots \\ \cdot & \cdot & \cdot & \cdot & \dots \end{bmatrix}$$

where P_{ij} is defined as:

$$\forall j \geq i-1, i \geq 1;$$

$$\mu_{\tilde{P}_{ij}}(x_{ij}) = \sup_{t \in \mathbb{R}^+} \left\{ \mu_{\tilde{S}}(t) \mid x_{ij} = \frac{\exp(-\lambda t)(\lambda t)^{j-i+1}}{(j-i+1)!} \right\}$$

The fuzzy-based controller adopted

The assumption in the design of the controller is that the server is switched on and remain active provided there are customers in the system awaiting service turns while it is switched off once all customers in the system had been served. It is possible to represent the state of the system as: (x, y) . In this case, $x = xt$ defines the server's state at time t . The arrivals in the network awaiting service turn at time t is $y = y(t)$. The value of y is stochastic in nature as it changes each time there is an arrival or a customer leaves the network due to service completion. Similar to this, x changes each time the server is switched on or off. With this, the average cost over a given period of time is given as:

$$g = \lim_{T \rightarrow \infty} \left[\frac{T}{T} \sum_{t=1}^T hs(t) + \frac{T}{T} \sum_{t=1}^T rk(t) + \frac{T}{T} \sum_{t=1}^T \frac{Rx1_{\{k(t-1)=0, k(t)=1\}}}{T} \right]$$

In this case, $1_{\{k(t-1)=0, k(t)=1\}}$ is 1 provided 'x' is 'true' and '0', if false. Consequently, the cost of switching on the server at time t is

$$Rx1_{\{k(t-1)=0, k(t)=1\}}$$

The cost, c is expressed as:



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$$c = h \sum_{i=1}^n s_i$$

In this case, 'i' denotes the ith consecutive unit of time in the state of the current, 'n' denotes the total time for which the server remain idle in state 'k' beginning from the time it was powered on while s_i denotes the arrivals awaiting service turns in the network at ith time. It might be optimal to switch on the server at any three cases, viz:

- a. When there is no switch on costs: In this case, it is optimal to switch on the server when a customer arrives. The existence of a switch-on cost could cause delays in turning on the server even when there are arrivals awaiting service turns in the network. This ensures cost reduction.
- b. When traffic intensity ρ is defined in the context of the function:

$$\rho = \lambda / \mu$$

With this, ρ denotes the average server's usage over an infinite time 1- ρ. This gives the fraction of time the server could remain idle. However, when ρ ≥ 1, arrivals increases infinitely.

- c. When h = 0, the idle server is switched off in order to achieve minimum cost g = 0. This happens even while arrivals increase as time T increases. In this scenario, it is assumed that the value of h is greater than zero. Consequently, the higher the value of h, the easier it becomes to decide when to switch on the server.

The rule-based inputs adopted include: c, h and ρ. Each of these is denoted with four linguistic values of NL, NP, PO and HP representing 'Nil', 'Nearly Positive', 'Positive' and 'Highly Positive' respectively. The integer value of each input value was made 0, 1, 2 and 3 for NL, NP, PO and HP respectively. The corresponding output for each rule combination was represented with dec. Decision regarding time to switch the server 'on' or 'off' is based on dec., represented with linguistic values: ON and OFF. This is given in table 1.

Table 1: Input values c, h and ρ with corresponding output value, dec.

Rule sequence	c	h	ρ	dec.
1	NL	NL	NL	OFF
2	NP	NL	NL	OFF
3	PO	NL	NL	OFF
4	HP	NL	NL	OFF
5	NL	NP	NL	OFF
6	NP	NP	NL	OFF
7	PO	NP	NL	ON
8	HP	NP	NL	ON
9	NL	PO	NL	OFF
10	NP	PO	NL	ON
11	PO	PO	NL	ON
12	HP	PO	NL	ON
13	NL	HP	NL	ON
14	NP	HP	NL	ON
15	PO	HP	NL	ON
16	HP	HP	NL	ON
⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮
55	PO	NP	HP	ON
56	HP	NP	HP	ON

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57	NL	PO	HP	ON
58	NP	PO	HP	ON
59	PO	PO	HP	ON
60	HP	PO	HP	ON
61	NL	HP	HP	ON
62	NP	HP	HP	ON
63	PO	HP	HP	ON
64	HP	HP	HP	ON

The decision (*dec.*) to switch ON or OFF is based on these input values. For instance, *dec.* is ON if input sum is greater than 3; if the sum is less than or equal to 2, the *dec.* is ON. The mathematical weight of rule 1, for instance is NL+NL+NL which is equivalent to $0 + 0 + 0 = 0$. This sum, i.e. 0 is less than 2 and therefore the *dec.* is OFF. In addition, rule 7 for example is PO + NP + NL which is equivalent to $2+1+0$. The mathematical weight is 3, which is greater than 2. Hence the *dec.* is ON. In most fuzzy applications, membership function for input variables is either triangular or trapezoidal. Functional forms of inputs' membership functions often depend on the corresponding inputs. The interval $[0, +\infty)$ is the domain of the accumulated cost c . Hence when $c \geq R$, then c is of the fuzzy set HP. With this, HP for c is of a trapezoidal membership function. The membership functions of NL, NP and PO for input value c are triangular in form with values distributed in the range of '0' and 'R'. To derive normalized values for input value c , observed values are multiplied by scaling factor $6/R$. Figure 3 is the membership functions of normalized input value c .

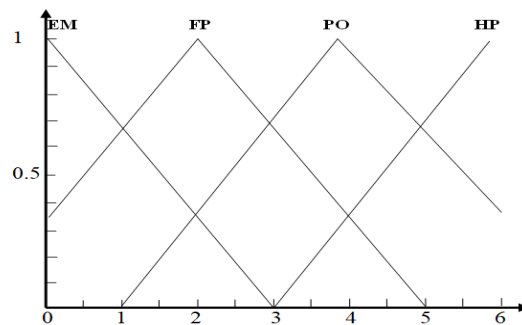


Figure 3: Membership functions for input c .

If $h \geq R$, then h has an HP set. In this case, when $h \rightarrow 0$, then h is of the set NL. When all customers had been served, the server is switched off. At this instance, the server is off during n unit time, i.e. $i = 1, 2, \dots, n$. A new arrival corresponds to a time unit when $s_i = s_{i+1} + 1$.

is proportional to ρ . The membership functions for input ρ is given in figure 4.

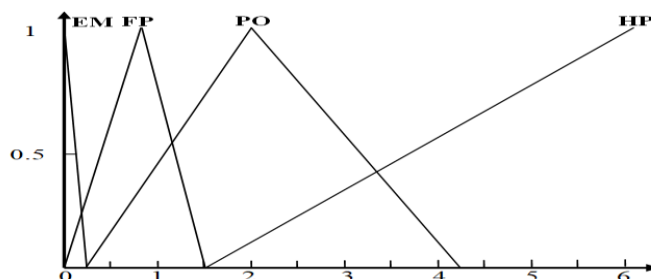


Figure 4: Membership functions for input ρ

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Membership functions in terms of input values for h are the same for ρ . The only difference is that while input values for ρ are bounded, that of h are not. The membership functions for input h are given in figure 5.

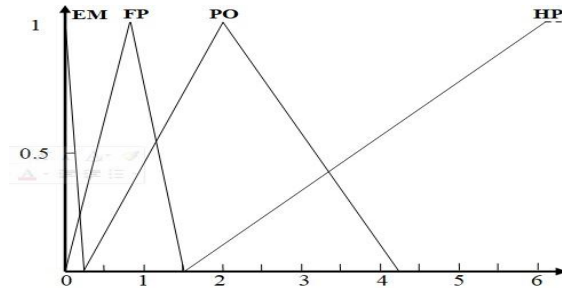


Figure 5: Membership functions for input h

Results of simulating the Markov queue system

The process of simulating the Markov queue system involves the use of computer models and programs to generate sample paths while also testing the attributes of the stochastic system. Consequently, at time zero, it is possible for a model to generate subsequent states as well as the corresponding transition epochs

The time of sojourn in state S_i has an exponential distribution λ_i with the probability of the network progressing from state S_i to S_j . In this case, the network is of a single state variable which ranges over the set $\{S_1, S_2, \dots\}$. If at time zero, the system is in state S_i , then it implies that the system will remain in this state for a random interval of time with an exponential distribution of mean $1/\lambda_i$.

From this state, it proceeds to state S_j with probability λ_{ij}/λ for some random time.

The system was simulated beginning with an initial state $k=0, s=0$ and $c=0$, while employing the fuzzy control at each decision epoch. The first 200 time units were considered the warm-up period for the network. The evolution of k and s for the first 1,500 time units is given in figure 6.

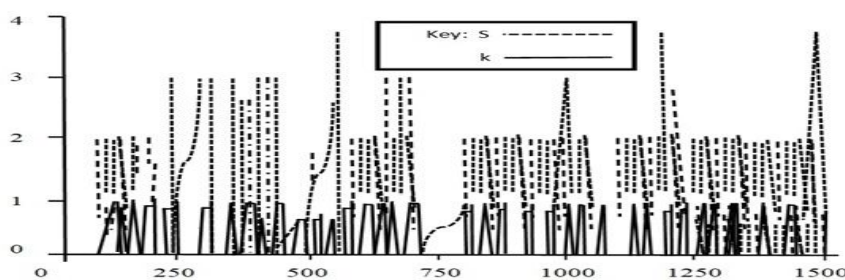


Figure 6: Evolution of k, s



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From figure 6, it is obvious that an idle server is not immediately turned on to begin service after an arrival occurs in the network but rather after the arrival of multiple customers. This shows the hystericity of the system, thereby establishing a direct relationship between the numbers of arrivals and possible number of times the server is switched on. This is an interplay in the number of times the server is switched on as a function of s for a period of 20,000 time units. With the hysteric property of the system, the server is often turned on when there are customers in the system. For a server with capacity $r \geq 1$, the system takes r customers per service. However, if at the end of service, less than r arrivals are waiting to be served, an idle period is activated. Consequently, the server proceeds on vacation. The server resumes service only when the queue size is equal to or greater than r . In this case, service is started when the queue crosses N from below and turns off when it crosses r from above. The input process is an orderly stationary Poisson process of $\{r_n; n \in \mathbb{N}\}$ with intensity $\lambda > 0$. While r_n is the arrival rate of the N^{th} customer, $N(t)$ is the associated counting process. Similarly, if σ_n denotes the service time of N^{th} batch of customers, it is assumed that

$\{\sigma_n; n \in \mathbb{N}\}$ and $\{r_n; n \in \mathbb{N}\}$ are independent. Let $T_n (T_0 = 0)$, represent the time of the N^{th} service completion, If queue size is greater than r , then

$$T_{n+1} - T_n = \sigma_n \quad \text{but if the queue size is less than } r,$$

$$T_{n+1} - T_n = (\text{time for the queue to reach } N) + \sigma_n$$

Similarly, if V_n denotes arrivals in the system during the N^{th} service σ_n and $Q(t)$ represent the size of the queue at any time t . If at time T_n , at least r customers are waiting to be served, then the

server takes a batch of r customers. The service time σ_n in this case is spread as a probability distribution function β having a finite first moment b . If less than r arrivals are waiting to be served at time T_n , then an idle period begins.

Conclusion

The method proposed is fuzzy-driven and it had proven efficient based on simulation results in the study. This implies that it could be adequately adapted in the efficient control of admissions in an M/M/1 queues with server vacations. This could also be applied in the management of network traffic and telecommunications control systems since admission control in these cases are of threshold type.

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